AP® CALCULUS BC
2008 SCORING GUIDELINES (Form B)

Question 1

A particle moving along a curve in the xy-plane has position $(x(t), y(t))$ at time $t \geq 0$ with

$$\frac{dx}{dt} = \sqrt{3t} \text{ and } \frac{dy}{dt} = 3\cos\left(\frac{t^2}{2}\right).$$

The particle is at position $(1, 5)$ at time $t = 4$.

(a) Find the acceleration vector at time $t = 4$.

(b) Find the y-coordinate of the position of the particle at time $t = 0$.

(c) On the interval $0 \leq t \leq 4$, at what time does the speed of the particle first reach 3.5 ?

(d) Find the total distance traveled by the particle over the time interval $0 \leq t \leq 4$.

(a) $a(4) = \langle x''(4), y''(4) \rangle = \langle 0.433, -11.872 \rangle$	1 : answer
(b) $y(0) = 5 + \int_4^0 3\cos\left(\frac{t^2}{2}\right) dt = 1.600$ or 1.601	$3 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{uses } y(4) = 5 \\ 1 : \text{answer} \end{cases}$
(c) Speed $= \sqrt{(x'(t))^2 + (y'(t))^2}$ $= \sqrt{3t + 9\cos^2\left(\frac{t^2}{2}\right)} = 3.5$ The particle first reaches this speed when $t = 2.225$ or 2.226.	$3 : \begin{cases} 1 : \text{expression for speed} \\ 1 : \text{equation} \\ 1 : \text{answer} \end{cases}$
(d) $\int_0^4 \sqrt{3t + 9\cos^2\left(\frac{t^2}{2}\right)}\, dt = 13.182$	$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

Form B
BC 1
1A1

CALCULUS BC

SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.

Work for problem 1(a)

acceleration = $\langle x''(t), y''(t)\rangle$

$x''(t) = \frac{3}{2}(3t)^{-1/2} = \frac{3}{2}((3)(4))^{-1/2} = 0.433$

$y''(t) = -11.872$ $\quad$ acceleration vector : $\langle 0.433, -11.872\rangle$

Work for problem 1(b)

$y(0) = y(4) + \int_4^0 3\cos\left(\frac{t^2}{2}\right)dt$

$y(0) = 5 + \int_4^0 3\cos\left(\frac{t^2}{2}\right)dt = 1.601$

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Continue problem 1 on page 5.

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Work for problem 1(c)

speed $= \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$

$\sqrt{3t + 9\cos^2\left(\frac{t^2}{2}\right)} = 3.5$

$\boxed{t = 2.226}$

Work for problem 1(d)

$\int_0^4 \sqrt{3t + 9\cos^2\left(\frac{t^2}{2}\right)}\, dt = \boxed{13.182}$

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Form B
BC1
1B1

CALCULUS BC

SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.

Work for problem 1(a)

$\frac{dx}{dt} = \sqrt{3t}$ $\qquad \frac{dy}{dt} = 3\cos\left(\frac{t^2}{2}\right)$

acceleration: $(x''(t), y''(t))$

$x''(t) = \sqrt{3} \times \frac{1}{2} \times \frac{1}{\sqrt{t}}$ $\qquad y''(t) = -3\sin\left(\frac{t^2}{2}\right) \times \frac{1}{2} \times 2t$

$= \frac{\sqrt{3}}{2\sqrt{t}}$ $\qquad = -3t\sin\left(\frac{t^2}{2}\right)$

$t=4$

$x''(4) = \frac{\sqrt{3}}{2\sqrt{4}}$ $\qquad y''(4) = -3\sin\left(\frac{4^2}{2}\right)$

$= \frac{\sqrt{3}}{4}$ $\qquad = -3\sin 8$

≈ 0.433 $\qquad \approx -2.968$

$\boxed{(0.433, -2.968)}$

Work for problem 1(b)

$\frac{dx}{dt} = \sqrt{3t}$ $\qquad \frac{dy}{dt} = 3\cos\left(\frac{t^2}{2}\right)$

$\int \frac{dx}{dt}\,dt = \int \sqrt{3t}\,dt$ $\qquad \int \frac{dy}{dt}\,dt = \int 3\cos\left(\frac{t^2}{2}\right)dt$

$x = \sqrt{3}\,t^{\frac{3}{2}} \times \frac{2}{3} + C$ $\qquad y = \int 3\cos\left(\frac{t^2}{2}\right)dt$

$\int_0^4 3\cos\left(\frac{t^2}{2}\right)dt = y(4) - y(0)$

$= 5 - y(0)$

$y(0) = 5 - \int_0^4 3\cos\left(\frac{t^2}{2}\right)dt$

$\approx 5 - 3.399$

$= \boxed{1.601}$

Continue problem 1 on page [redacted]

Work for problem 1(c)

$$\text{speed} = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$$

$$= \sqrt{(\sqrt{3t})^2 + \left(3\cos\left(\frac{t^2}{2}\right)\right)^2}$$

$$= \sqrt{3t + 9\cos^2\left(\frac{t^2}{2}\right)}$$

$$= 3.5 \qquad 0 \le t \le 4$$

$$t = \boxed{2.226}$$

Work for problem 1(d)

$$\sqrt{\left(\int_0^4 \sqrt{3t}\, dt\right)^2 + \left(\int_0^4 3\cos\left(\frac{t^2}{2}\right) dt\right)^2}$$

$$\approx \sqrt{(9.238)^2 + (3.399)^2}$$

$$\approx \boxed{9.843}$$

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Form B
BC1
1C1

CALCULUS BC

SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.

Work for problem 1(a)

0.14

acceleration = (0.14, −11.87)

$(V)' = a$

$\left(\frac{dx}{dt}\right)' = (\sqrt{3t})'$

$\left((3t)^{\frac{1}{2}}\right)' = \frac{1}{2}(3t)^{-\frac{1}{2}} \cdot 3$

$a = \frac{3}{2\sqrt{3t}}$; (t=4) $a_x = \frac{3}{2\sqrt{12}} \approx 0.14$

$\left(\frac{dy}{dt}\right)' = \left(3\cos\left(\frac{t^2}{2}\right)\right)'$ $-3t\sin\left(\frac{t^2}{2}\right)$

t = 4, $a_y = -12\sin(8)$

Acceleration = (0.14, −11.87)

Work for problem 1(b)

y position $= \int 3\cos\left(\frac{t^2}{2}\right) dt =$

$= 3\int \cos\left(\frac{t^2}{2}\right) dt$

$= 3\left(\frac{\sin\left(\frac{t^2}{2}\right)}{t}\right) + C$

t = 0, such that

Continue problem 1 on page 5.

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Work for problem 1(c)

speed : $\sqrt{(\sqrt{3t})^2 + 3\cos(\frac{t^2}{2})^2}$

$3.5 = \sqrt{(\sqrt{3t})^2 + 3\cos(\frac{t^2}{2})^2}$

$12.25 = 3t + (3\cos(\frac{t^2}{2}))^2$

$\Leftrightarrow 12.25 = 3t + 9\cos^2(\frac{t^2}{2})$

graph: $3t + 9\cos^2(\frac{t^2}{2}) - 12.25$

at 2.22 second, the speed of the particle first reach 3.5

Work for problem 1(d)

Total dist. $= \int_0^4 \sqrt{(\sqrt{3t})^2 + (3\cos(\frac{t^2}{2}))^2}\, dt$

$\Leftrightarrow \int_0^4 \sqrt{3t^2 + 9\cos^2(\frac{t^2}{2})}$

$\simeq 17.48$

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Question 1

Sample: 1A
Score: 9

The student earned all 9 points.

Sample: 1B
Score: 6

The student earned 6 points: no points in part (a), 3 points in part (b), 3 points in part (c), and no points in part (d). The student presents correct work in parts (b) and (c). In part (a) the student has the correct value of $x''(4)$ but loses a factor of t in $y''(4)$, and thus the response did not earn the point. In part (d) the student does not integrate the speed in order to determine the total distance traveled.

Sample: 1C
Score: 4

The student earned 4 points: no points in part (a), 1 point in part (b), 2 points in part (c), and 1 point in part (d). In part (a) the student is not able to determine $x''(4)$. In part (b) the student has the correct integrand and earned 1 point. The student does not use the initial condition of $y(4) = 5$ so was not eligible for the answer point. In part (c) the student finds the correct equation and earned the first 2 points. The answer is only given to two decimal places, so the third point was not earned. In part (d) the student has the correct integral to determine the distance traveled and earned the first point; however, the answer is not correct, so the answer point was not earned.

Question 2

For time $t \geq 0$ hours, let $r(t) = 120\left(1 - e^{-10t^2}\right)$ represent the speed, in kilometers per hour, at which a car travels along a straight road. The number of liters of gasoline used by the car to travel x kilometers is modeled by $g(x) = 0.05x\left(1 - e^{-x/2}\right)$.

(a) How many kilometers does the car travel during the first 2 hours?

(b) Find the rate of change with respect to time of the number of liters of gasoline used by the car when $t = 2$ hours. Indicate units of measure.

(c) How many liters of gasoline have been used by the car when it reaches a speed of 80 kilometers per hour?

(a) $\int_0^2 r(t)\,dt = 206.370$ kilometers

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(b) $\dfrac{dg}{dt} = \dfrac{dg}{dx} \cdot \dfrac{dx}{dt}; \quad \dfrac{dx}{dt} = r(t)$

$$\left.\frac{dg}{dt}\right|_{t=2} = \left.\frac{dg}{dx}\right|_{x=206.370} \cdot r(2)$$

$$= (0.050)(120) = 6 \text{ liters/hour}$$

$3 : \begin{cases} 2 : \text{uses chain rule} \\ 1 : \text{answer with units} \end{cases}$

(c) Let T be the time at which the car's speed reaches 80 kilometers per hour.

Then, $r(T) = 80$ or $T = 0.331453$ hours.

At time T, the car has gone

$$x(T) = \int_0^T r(t)\,dt = 10.794097 \text{ kilometers}$$

and has consumed $g(x(T)) = 0.537$ liters of gasoline.

$4 : \begin{cases} 1 : \text{equation } r(t) = 80 \\ 2 : \text{distance integral} \\ 1 : \text{answer} \end{cases}$

Form B
AB/BC 2
2A$_1$

Work for problem 2(a)

Distance travelled by car during first 2 hours $= \int_0^2 r(t)\,dt$

$= \int_0^2 120(1-e^{-10t^2})\,dt$

$= 206.370$ km

Work for problem 2(b)

$g(x) = 0.05x(1-e^{-x/2})$

Rate of change w.r.t. ~~x~~ time of $g(x) = \frac{dg(x)}{dt} = 0.05(1-e^{-x/2})r(t) + 0.05x(\frac{1}{2}e^{-x/2})r(t)$

$\therefore$ At time $t = 2$ hours, Rate of change w.r.t time of $g(x)$

$= 0.05(1 - e^{(206.370/2)})r(2) + 0.05(206.370)(\frac{1}{2}e^{(-206.370/2)})r(2)$

$= 6$ L/hr.

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Continue problem 2 on page 7.

Work for problem 2(c)

$r(t) = 80$ km/hr $\Rightarrow 120(1 - e^{-10t^2}) = 80$

$\therefore t = 0.331$ hr

$\therefore$ Distance travelled $= \int_0^{0.331} r(t)\,dt$

$= \int_0^{0.331} 120(1 - e^{-10t^2})\,dt$

$= 10.794$ km

$\therefore$ Liters of gasoline used $= g(10.794)$

$= 0.05(10.794)(1 - e^{(10.794/2)})$

$= 0.537$ L

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Form B.
AB/BC 2
2B1

Work for problem 2(a)

$r(t) = 120(1 - e^{-10t^2})$

$\int_0^2 r(t) = 206.37$

$\boxed{A: 206.37 \text{ km}}$

Work for problem 2(b)

$r(t) = \frac{dr}{dt} = 120(1 - e^{-10t^2})$

$R(t) = \int 120(1 - e^{-10t^2}) \text{. km}$

$g(R(t)) = 0.05x(1 - e^{-x/2})$

$g'(R(t)) = 0.05e^{-\frac{x}{2}} \cdot (e^{\frac{x}{2}} + 0.5(x-2))$

when $t = 2$;
$x = 206.37$ km

$g'(x) = 0.05$

$\boxed{A: 0.05 \text{ liters per km}}$

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Continue problem 2 on page 7.

Work for problem 2(c)

$r(t) = 120\left(1 - e^{-10t^2}\right) = 80.$

$t = \cancel{-0.331453}$ or 0.331453

$R(t) = \int_0^{0.331453} r(t)$

$= 10.7941$ km.

$\rightarrow g(10.7941) = 0.53726$

$\boxed{\text{A: } 0.537 \text{ liters}}$

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Form B
AB/BC 2
2C1

Work for problem 2(a)

the distance traveled = $\int_0^2 r(t)\,dt$

$= \int_0^2 120(1-e^{-10t^2})\,dt = 206.37$ km

Work for problem 2(b)

$\frac{g'(x)}{r'(t)} =$

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Continue problem 2 on page 7.

Work for problem 2(c)

at the speed of 80

$$80 = 120(1-e^{-10t^2})$$

we find (t) and subsitet it in the result found in part ~~a~~ (b)

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AP® CALCULUS BC
2008 SCORING COMMENTARY (Form B)

Question 2

Sample: 2A
Score: 9

The student earned all 9 points.

Sample: 2B
Score: 6

The student earned 6 points: 2 points in part (a), no points in part (b), and 4 points in part (c). The student presents correct work in parts (a) and (c). In part (b) the student attempts to use the chain rule but does not put together the correct pieces necessary to answer the question.

Sample: 2C
Score: 3

The student earned 3 points: 2 points in part (a), no points in part (b), and 1 point in part (c). The student presents correct work in part (a). No points were earned in part (b). In part (c) the student sets $r(t) = 80$ and earned the first point. Since the student does not solve the equation for t, the response did not earn the remaining points.

AP® CALCULUS BC
2008 SCORING GUIDELINES (Form B)

Question 3

Distance from the river's edge (feet)	0	8	14	22	24
Depth of the water (feet)	0	7	8	2	0

A scientist measures the depth of the Doe River at Picnic Point. The river is 24 feet wide at this location. The measurements are taken in a straight line perpendicular to the edge of the river. The data are shown in the table above. The velocity of the water at Picnic Point, in feet per minute, is modeled by $v(t) = 16 + 2\sin\left(\sqrt{t+10}\right)$ for $0 \le t \le 120$ minutes.

(a) Use a trapezoidal sum with the four subintervals indicated by the data in the table to approximate the area of the cross section of the river at Picnic Point, in square feet. Show the computations that lead to your answer.

(b) The volumetric flow at a location along the river is the product of the cross-sectional area and the velocity of the water at that location. Use your approximation from part (a) to estimate the average value of the volumetric flow at Picnic Point, in cubic feet per minute, from $t = 0$ to $t = 120$ minutes.

(c) The scientist proposes the function f, given by $f(x) = 8\sin\left(\frac{\pi x}{24}\right)$, as a model for the depth of the water, in feet, at Picnic Point x feet from the river's edge. Find the area of the cross section of the river at Picnic Point based on this model.

(d) Recall that the volumetric flow is the product of the cross-sectional area and the velocity of the water at a location. To prevent flooding, water must be diverted if the average value of the volumetric flow at Picnic Point exceeds 2100 cubic feet per minute for a 20-minute period. Using your answer from part (c), find the average value of the volumetric flow during the time interval $40 \le t \le 60$ minutes. Does this value indicate that the water must be diverted?

(a) $\frac{(0+7)}{2}\cdot 8 + \frac{(7+8)}{2}\cdot 6 + \frac{(8+2)}{2}\cdot 8 + \frac{(2+0)}{2}\cdot 2$
$= 115 \text{ ft}^2$

1 : trapezoidal approximation

(b) $\frac{1}{120}\int_0^{120} 115v(t)\,dt$
$= 1807.169$ or 1807.170 ft^3/min

3 : { 1 : limits and average value constant; 1 : integrand; 1 : answer }

(c) $\int_0^{24} 8\sin\left(\frac{\pi x}{24}\right) dx = 122.230$ or 122.231 ft^2

2 : { 1 : integra1; 1 : answer }

(d) Let C be the cross-sectional area approximation from part (c). The average volumetric flow is
$\frac{1}{20}\int_{40}^{60} C\cdot v(t)\,dt = 2181.912$ or 2181.913 ft^3/min.

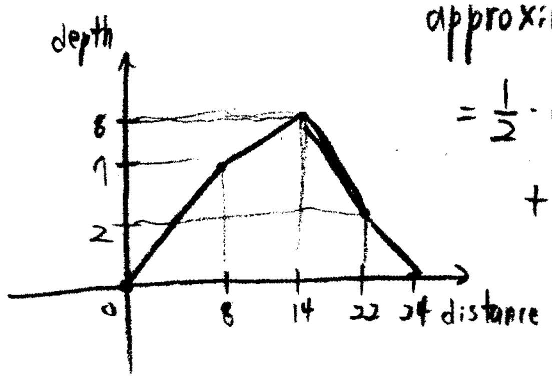
Yes, water must be diverted since the average volumetric flow for this 20-minute period exceeds 2100 ft^3/min.

3 : { 1 : volumetric flow integral; 1 : average volumetric flow; 1 : answer with reason }

Form B
AB/BC 3
3A1

Distance from the river's edge (feet)	0	8	14	22	24
Depth of the water (feet)	0	7	8	2	0

Work for problem 3(a)



approximation of area using trapezoidal sum

$$= \frac{1}{2}\cdot(8)\cdot 7 + \frac{1}{2}\underbrace{(7+8)}_{=15}\cdot\underbrace{(14-8)}_{=6}$$

$$+ \frac{1}{2}\cdot\underbrace{(8+2)}_{=10}\cdot\underbrace{(22-14)}_{=8} + \frac{1}{2}\cdot 2\cdot\underbrace{(24-22)}_{=2}$$

$$= 4\cdot 7 + 3\cdot 15 + 5\cdot 8 + 2$$

$$= 28 + 45 + 40 + 2$$

$$= 73 + 42 = 115$$

$$\underline{115\ (ft)^2}$$

Work for problem 3(b)

Average value of volumetric flow at Picnic Point

$$= \frac{1}{120-0}\left(\int_0^{120} v(t)\,dt\right)\cdot 115 = \frac{115}{120}\int_0^{120}\left(16 + 2\sin\left(\sqrt{t+10}\right)\right)dt$$

$$= \underline{1809.16491\ (ft)^3/min}$$

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Continue problem 3 on page 9.

3A2

Work for problem 3(c)

$$\text{Area} = \int_0^{24} 8\sin\left(\frac{\pi x}{24}\right) dx$$

$\cos\pi = -1$

$\cos(0) = 1$

$$= \left[-\frac{24}{\pi}\cdot 8\cos\left(\frac{\pi x}{24}\right)\right]_0^{24} = -\frac{24\cdot 8}{\pi}\cos(\pi) + \frac{24\cdot 8}{\pi}\cos(0)$$

$$= -\frac{24\cdot 8}{\pi}\cdot(-1) + \frac{24\cdot 8}{\pi}\cdot(1)$$

$$= 2\cdot\frac{24\cdot 8}{\pi} = \underline{122.23099 (ft)^2}$$

Work for problem 3(d)

Average value of volumetric flow during $40 \le t \le 60$

$$= \frac{1}{60-40}\left(\int_{40}^{60}\left(16 + 2\sin\left(\sqrt{t+10}\right)\right)dt\right)\cdot(122.23099)$$

$$= \frac{122.23099}{20}\cdot\int_{40}^{60}\left(16 + 2\sin\left(\sqrt{t+10}\right)\right)dt$$

$$= \underline{2181.91253\ (ft)^3/min} > 2100\ (ft)^3/min$$

Thus, water must be diverted.

END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

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Form B
AB/BC 3
3B₁

Distance from the river's edge (feet)	0	8	14	22	24
Depth of the water (feet)	0	7	8	2	0

Work for problem 3(a)

24ft

Traperoidal sum = $\frac{\text{Right sum} + \text{left sum}}{2}$

Right sum = $(2\cdot 0)+(8\cdot 2)+(6\cdot 8)+(8\cdot 7)$

$= 120$

left sum = $(8\cdot 0)+(6\cdot 7)+(8\cdot 8)+(2\cdot 2)$

$= 110$

Traperoidal sum = $\frac{120+110}{2} = \boxed{115 \text{ square feet}}$

Work for problem 3(b)

Flow = $115 \cdot v(t)$

$= 115 \cdot v(120)$

$= 115 \cdot 14.1627263406$

$= \boxed{1628.714 \text{ cubic feet per minute}}$

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Continue problem 3 on page 9.

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Work for problem 3(c)

(122.2309963)

$$\int_0^{24} f(x)\,dx = \boxed{122.231 \text{ square feet}}$$

Work for problem 3(d)

$\frac{1}{20}\int$ Volumetric flow > 2100 ft^3/min $\Rightarrow$ must be diverted.

$$\frac{1}{60-40}\int_{40}^{60} (122.231)(V(t))\,dt$$

$$= \frac{1}{20}(43638.25299)$$

$=$ 2181.913 ft^3/min. $\Rightarrow$ Yes, this value indicates that the water must be diverted.

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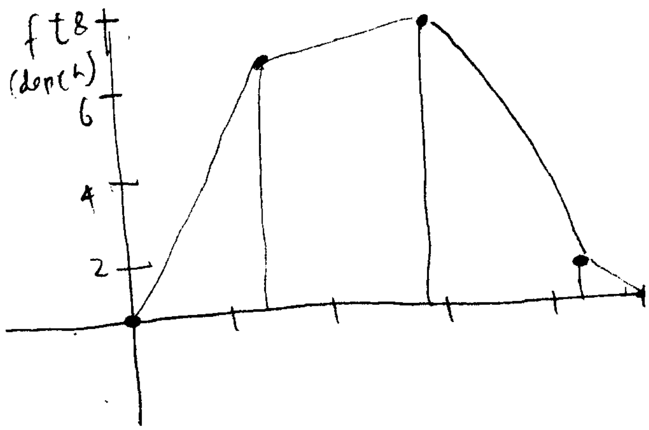
END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

Form B
AB/BC 3
3C1

Distance from the river's edge (feet)	0	8	14	22	24
Depth of the water (feet)	0	7	8	2	0

Work for problem 3(a)



$\frac{1}{2}(b_1+b_2)h + \frac{1}{2}(b_2+b_3)h + \frac{1}{2}(b_3+b_4)h$

$+ \frac{1}{2}(b_4+b_5)h.$

$= \frac{1}{2}(0+7)8 + \frac{1}{2}(7+8)6$

$+ \frac{1}{2}(8+2)\times 8 + \frac{1}{2}(2+0)\times 2$

$= 115\ ft^2$

$ft^2 \times ft/m$

Work for problem 3(b)

$$\text{Avg} = \frac{\text{approx Area} \times V(0) + \text{approx Area} \times V(120)}{\cancel{120}}$$

$$= \frac{115 \times (16+2\sin(\sqrt{0+10})) + 115\,(16+2\sin(\sqrt{120+10}))}{\cancel{120}}$$

$= \cancel{28.9}\ ft^3/\text{minute}$

3463

Continue problem 3 on page 9.

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Work for problem 3(c)

$$\text{area} = \int_0^{24} f(x)$$

$$\int_0^{24} 8\sin\left(\frac{\pi}{24}x\right)dx = -8 \times \frac{24}{\pi} \times \cos\left(\frac{\pi}{24}x\right)\Big|_0^{24}$$

$$= 122.231\ ft^2$$

Work for problem 3(d)

$$\text{Area} \times V(40) + \text{Area} \times V(60)$$

$$= 122.231 \times (16 + 2\sin\sqrt{40+10}) + 122.231 \times (16 + 2\sin\sqrt{60+10})$$

$$= 4293.87\ ft^3/min$$

$\therefore$ must be diverted.

END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

Question 3

Sample: 3A
Score: 9

The student earned all 9 points.

Sample: 3B
Score: 6

The student earned 6 points: 1 point in part (a), no points in part (b), 2 points in part (c), and 3 points in part (d). The student presents correct work in parts (a), (c), and (d). In part (b) the student does not produce an integral, which is needed in order to find the average value of volumetric flow.

Sample: 3C
Score: 3

The student earned 3 points: 1 point in part (a), no points in part (b), 2 points in part (c), and no points in part (d). The student presents correct work in parts (a) and (c). In part (b) the student does not produce an integral to find the average value and thus did not earn any points. In part (d) the student also does not produce an integral and did not earn any points. Although the student's statement that the water "must be diverted" is true, the student does not present enough correct calculus work leading up to the answer to earn the answer point.

Question 4

Let f be the function given by $f(x) = kx^2 - x^3$, where k is a positive constant. Let R be the region in the first quadrant bounded by the graph of f and the x-axis.

(a) Find all values of the constant k for which the area of R equals 2.

(b) For $k > 0$, write, but do not evaluate, an integral expression in terms of k for the volume of the solid generated when R is rotated about the x-axis.

(c) For $k > 0$, write, but do not evaluate, an expression in terms of k, involving one or more integrals, that gives the perimeter of R.

(a) For $x \ge 0$, $f(x) = x^2(k - x) \ge 0$ if $0 \le x \le k$

$$\int_0^k \left(kx^2 - x^3\right) dx = \left(\frac{k}{3}x^3 - \frac{1}{4}x^4\right)\Bigg|_{x=0}^{x=k} = \frac{k^4}{12}$$

Area $= \dfrac{k^4}{12} = 2$; $k = \sqrt[4]{24}$

$$4 : \begin{cases} 1 : \text{integral} \\ 1 : \text{antiderivative} \\ 1 : \text{value of integral} \\ 1 : \text{answer} \end{cases}$$

(b) Volume $= \pi \displaystyle\int_0^k \left(kx^2 - x^3\right)^2 dx$

$$2 : \begin{cases} 1 : \text{ integrand} \\ 1 : \text{ limits and constant} \end{cases}$$

(c) Perimeter $= k + \displaystyle\int_0^k \sqrt{1 + \left(2kx - 3x^2\right)^2}\, dx$

$$3 : \begin{cases} 1 : \int_0^k \sqrt{1 + (f'(x))^2}\, dx \\ 1 : \text{uses } f'(x) = 2kx - 3x^2 \text{ in integrand} \\ 1 : \text{answer} \end{cases}$$

Form B
BC 4
4A$_1$

NO CALCULATOR ALLOWED

CALCULUS BC

SECTION II, Part B

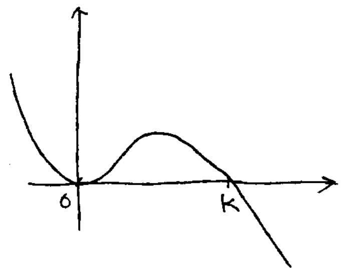
Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.

Work for problem 4(a)

$f(x) = kx^2 - x^3$
$\quad = x^2(k-x) \qquad (k>0)$



$R = \int_0^k f(x)\,dx$

$= \int_0^k (kx^2 - x^3)\,dx$

$= \frac{k}{3}x^3 - \frac{1}{4}x^4 \Big|_0^k$

$= \frac{1}{3}k^4 - \frac{1}{4}k^4 = \frac{1}{12}k^4 = 2$

$k^4 = 24$

$k = \sqrt[4]{24}$

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Continue problem 4 on page 11.

NO CALCULATOR ALLOWED

Work for problem 4(b)

$$\pi\int_0^k f(x)^2\,dx = \pi\int_0^k (kx^2-x^3)^2\,dx$$

Work for problem 4(c)

$$k+\int_0^k dl = k+\int_0^k \sqrt{1+(2kx-3x^2)^2}\,dx$$

$f'(x)=2kx-3x^2$

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GO ON TO THE NEXT PAGE.

Form B
BC 4
4B 1

NO CALCULATOR ALLOWED

CALCULUS BC

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.

Work for problem 4(a)

Find the values of x where ~~though~~ $f(x) = 0$.

$f(x) = x^2(k-x)$.

Thus, $x = 0$ or k.

$R = \int_0^k f(x)\,dx$.

$= \frac{1}{3}kx^3 - \frac{1}{4}x^4 \Big|_0^k$

$= \frac{1}{3}k^4 - \frac{1}{4}k^4 - 0 = 2$.

$\frac{1}{12}k^4 = 2$

$k^4 = 24$

$k = \sqrt[4]{24}$ or $-\sqrt[4]{24}$

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Continue problem 4 on page 11.

4B2

Work for problem 4(b)

$$\pi \int_0^k (kx^2 - x^3)\,dx.$$

Work for problem 4(c)

The perimeter of R is the length of f(x) and the length of x-axis in the interval of (0, k).

That is: $k + \int_0^k \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$

$$= k + \int_0^k \sqrt{1+(2kx-3x^2)}\,dx$$

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Form B
BC 4
4C1

NO CALCULATOR ALLOWED

CALCULUS BC

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.

Work for problem 4(a)

$f(x) = kx^2 - x^3$

$\int f(x) = 2$

$y = kx^2 - x^3$

x-intercept: $0 = kx^2 - x^3$

$0 = x^2(k - x)$

$x = k$

$\int_0^k f(x) = 2$

$\int_0^k kx^2 - x^3 = 2$

$\left[\frac{1}{3}kx^3 - \frac{1}{4}x^4\right]_0^k = 2$

$\left(\frac{1}{3}k(k^3) - \frac{1}{4}k^4\right) - (0) = 2$

$\frac{1}{12}k^4 = 2$

$k^4 = 24$

$k = \sqrt[4]{24}$

$= 2\sqrt[4]{6}$

$= \pm 2\sqrt{6}$

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Continue problem 4 on page 11.

Work for problem 4(b)

if $k>0$, $k = -2\sqrt{6}$ (rejected)(not applicable)

$$V = \pi \int y^2 \, dx$$

$$= \pi \int (kx^2 - x^3)^2 \, dx$$

$$= \pi \int k^2x^4 - 2kx^5 + x^6 \, dx$$

$$= \pi \left[\tfrac{1}{5}k^2x^5 - \tfrac{2}{6}kx^6 + \tfrac{1}{7}x^6 \right]$$

$$= \pi \left[\tfrac{1}{5}k^2x^5 - \tfrac{1}{3}kx^6 + \tfrac{1}{7}x^6 \right]$$

Work for problem 4(c)

Perimeter of R = k + y-intercept + length of curve

y-intercept: $y = 0 - 0 = 0$

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Question 4

Sample: 4A
Score: 9

The student earned all 9 points.

Sample: 4B
Score: 6

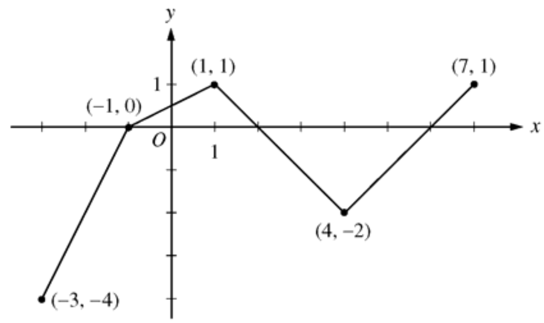
The student earned 6 points: 3 points in part (a), 1 point in part (b), and 2 points in part (c). In part (a) the student presents correct work, but since $k > 0$ is specified in the problem, the answer point was not earned. In part (b) the student earned 1 point for the limits and constant, but because the square is not included on the integrand, the integrand point was not earned. In part (c) the student does not include the square on the integrand, and so the answer point was not earned.

Sample: 4C
Score: 4

The student earned 4 points: 3 points in part (a), 1 point in part (b), and no points in part (c). In part (a) the student presents correct work, but since $k > 0$ is specified in the problem, the answer point was not earned. In part (b) the student has a correct integrand but does not include any limits on the integral. In part (c) the student does not show enough calculus work to be eligible for any points.

AP® CALCULUS BC
2008 SCORING GUIDELINES (Form B)

Question 5



Graph of g'

Let g be a continuous function with $g(2) = 5$. The graph of the piecewise-linear function g', the derivative of g, is shown above for $-3 \le x \le 7$.

(a) Find the x-coordinate of all points of inflection of the graph of $y = g(x)$ for $-3 < x < 7$. Justify your answer.

(b) Find the absolute maximum value of g on the interval $-3 \le x \le 7$. Justify your answer.

(c) Find the average rate of change of $g(x)$ on the interval $-3 \le x \le 7$.

(d) Find the average rate of change of $g'(x)$ on the interval $-3 \le x \le 7$. Does the Mean Value Theorem applied on the interval $-3 \le x \le 7$ guarantee a value of c, for $-3 < c < 7$, such that $g''(c)$ is equal to this average rate of change? Why or why not?

(a) g' changes from increasing to decreasing at $x = 1$;
g' changes from decreasing to increasing at $x = 4$.

Points of inflection for the graph of $y = g(x)$ occur at $x = 1$ and $x = 4$.

$2 : \begin{cases} 1 : x\text{-values} \\ 1 : \text{justification} \end{cases}$

(b) The only sign change of g' from positive to negative in the interval is at $x = 2$.

$$g(-3) = 5 + \int_2^{-3} g'(x)\,dx = 5 + \left(-\frac{3}{2}\right) + 4 = \frac{15}{2}$$
$$g(2) = 5$$
$$g(7) = 5 + \int_2^{7} g'(x)\,dx = 5 + (-4) + \frac{1}{2} = \frac{3}{2}$$

The maximum value of g for $-3 \le x \le 7$ is $\frac{15}{2}$.

$3 : \begin{cases} 1 : \text{identifies } x = 2 \text{ as a candidate} \\ 1 : \text{considers endpoints} \\ 1 : \text{maximum value and justification} \end{cases}$

(c) $$\frac{g(7) - g(-3)}{7 - (-3)} = \frac{\frac{3}{2} - \frac{15}{2}}{10} = -\frac{3}{5}$$

$2 : \begin{cases} 1 : \text{difference quotient} \\ 1 : \text{answer} \end{cases}$

(d) $$\frac{g'(7) - g'(-3)}{7 - (-3)} = \frac{1 - (-4)}{10} = \frac{1}{2}$$

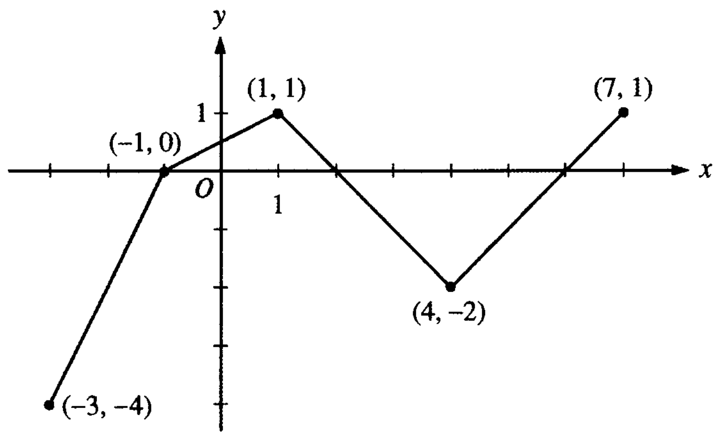
No, the MVT does *not* guarantee the existence of a value c with the stated properties because g' is not differentiable for at least one point in $-3 < x < 7$.

$2 : \begin{cases} 1 : \text{average value of } g'(x) \\ 1 : \text{answer “No” with reason} \end{cases}$

5 5 5 5 5

Form B
AB/BC 5
5A1

NO CALCULATOR ALLOWED



Graph of g'

Work for problem 5(a)

At points of inflection, g' should change from increasing to decreasing, or from decreasing to increasing.

Therefore, $x = 1, 4$

Work for problem 5(b)

$g'(x) = 0$ or the endpoints

$x=-3$: $\frac{7}{2} + 4 = \frac{15}{2}$

$x=-1$: $5 - 3 \times 1 \times \frac{1}{2} = \frac{7}{2}$

$x=2$: $g(2) = 5$

$x=6$: $g(6) = 5 - 4 \times 2 \times \frac{1}{2} = 1$

$x=7$: $g(7) = 1 + 1 \times 1 \times \frac{1}{2} = \frac{3}{2}$

therefore, the absolute maximum is $\frac{15}{2}$ when $x=-3$.

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Continue problem 5 on page 13.

NO CALCULATOR ALLOWED

Work for problem 5(c)

$$\frac{g(7)-g(-3)}{7-(-3)} = \frac{\frac{3}{2}-\frac{15}{2}}{10} = \frac{\frac{\cancel{-12}^{\,6}}{\cancel{2}}}{10} = -\frac{3}{5}$$

Work for problem 5(d)

$$\frac{g'(7)-g'(-3)}{7-(-3)} = \frac{1-(-4)}{10} = \frac{1}{2}$$

But Mean Value Theorem ~~[illegible]~~ doesn't guarantee a value of c such that $g''(c) = \frac{1}{2}$

because ~~[illegible]~~ function g' is not differentiable ~~[illegible]~~ at some points.

FoRm B
AB/BC 5
5B1

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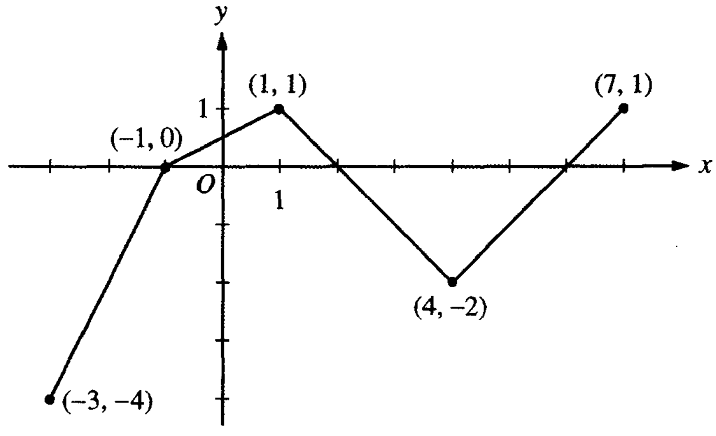

Graph of g'

Work for problem 5(a)

$\boxed{x=1, \quad x=4}$

Since inflection of the graph $(g(x))$ is the point that $g'(x)$ increasing become decrease or $g'(x)$ decreasing become increase.

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Work for problem 5(b)

$x=-3$

$g(-3)-4+\frac{3}{2}=5$

$\therefore g(-3)=\boxed{\frac{15}{2}}$

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Continue problem 5 on page 13.

NO CALCULATOR ALLOWED

Work for problem 5(c)

$$\frac{g(7)-g(-3)}{7-(-3)} = \frac{\frac{3}{2}-\frac{15}{2}}{10} = \frac{-6}{10} = \boxed{-0.6}$$

Meanwhile, $g(7) = g(2) - 4 + \frac{1}{2}$

$= 5 - 4 + \frac{1}{2}$

$= \frac{3}{2}$

$g(-3) = \frac{15}{2}$

Work for problem 5(d)

$$\frac{g'(7)-g'(-3)}{7-(-3)} = \frac{1-(-4)}{10} = \boxed{\frac{1}{2}}$$

No, it doesn't guarantee. Since $g'(x)$ is not ~~derivati~~ derivated at $x = -1, 1, 4$.

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Form B
AB/BC 5
5C1

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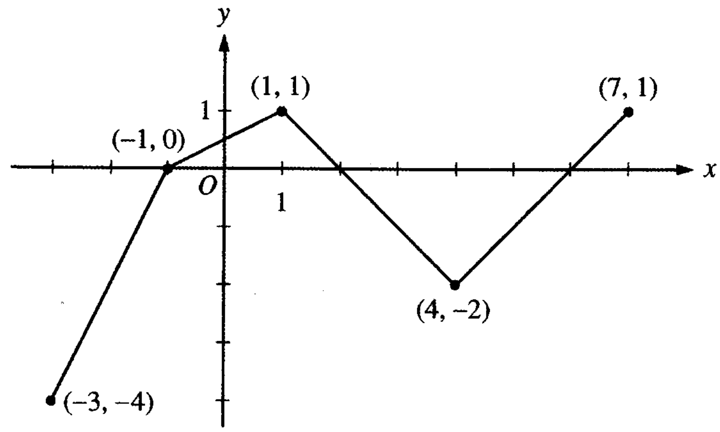

Graph of g'

Work for problem 5(a)

all points of inflection must have $f'(x)=0$ and change signs.

so $x=-1, 2$ and 6

Work for problem 5(b)

$\int_{-3}^{-1} g'(x)dx = 4$

$\int_{-1}^{2} g'(x)dx = 1.5$

$\int_{2}^{6} g'(x)dx = 4$

$\int_{6}^{7} g'(x)dx = 0.5$

since $g(2)=5$

so $g(-1)=3.5$

$g(-3)=7.5$

$g(6)=1$

$g(7)=1.5$

so the absolute maximum value of g on $-3 \le x \le 7$ is 7.5

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Continue problem 5 on page 13.

Work for problem 5(c)

average rate of change $= \dfrac{\int_{-3}^{7} g'(x)\,dx}{10}$

$= \dfrac{4+1.5+0.5+4}{10} = 1$

Work for problem 5(d)

average change of $g'(x) = \dfrac{2}{2} + \dfrac{\frac{1}{2}}{2} - \dfrac{1}{1} - \dfrac{1}{2} + \dfrac{1}{3} = \dfrac{1}{12}$

The Mean Value Theorem applied on the Interval $-3 \le x \le 7$ can not guarantee a value of c for $-3 < c < 7$ such that $g''(c)$ is equal to this average change. Since the rate of change is not continuous on the interval

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Question 5

Sample: 5A
Score: 9

The student earned all 9 points.

Sample: 5B
Score: 6

The student earned 6 points: 2 points in part (a), no points in part (b), 2 points in part (c), and 2 points in part (d). The student presents correct work in parts (a), (c), and (d). In part (b) the student does not identify $x = 2$ as a candidate, so the first point was not earned. The student finds the value of $g(-3)$ but does not find the value at the other endpoint, so the second point was not earned. The student did not earn the justification point since the work is not sufficient to state that $\frac{15}{2}$ is the maximum value.

Sample: 5C
Score: 4

The student earned 4 points: no points in part (a), 3 points in part (b), 1 point in part (c), and no points in part (d). In part (a) the student does not identify the correct values for the points of inflection. The student presents correct work in part (b). In part (c) the student uses the fact that the average rate of change is the average value of $g'(x)$ and presents a correct integral. The student makes an error in calculating the value of the integral so earned only 1 of the 2 points. In part (d) the student has an incorrect result for the average value of $g'(x)$, so the first point was not earned. Although the student declares that "[t]he Mean Value Theorem . . . can not guarantee a value of c" with the stated properties, the response includes the incorrect statement that "the rate of change is not continuous." Thus the student did not earn the second point.

Question 6

Let f be the function given by $f(x) = \dfrac{2x}{1+x^2}$.

(a) Write the first four nonzero terms and the general term of the Taylor series for f about $x = 0$.

(b) Does the series found in part (a), when evaluated at $x = 1$, converge to $f(1)$? Explain why or why not.

(c) The derivative of $\ln\left(1+x^2\right)$ is $\dfrac{2x}{1+x^2}$. Write the first four nonzero terms of the Taylor series for $\ln\left(1+x^2\right)$ about $x = 0$.

(d) Use the series found in part (c) to find a rational number A such that $\left|A - \ln\left(\frac{5}{4}\right)\right| < \frac{1}{100}$. Justify your answer.

(a)
$$\frac{1}{1-u} = 1 + u + u^2 + \cdots + u^n + \cdots$$
$$\frac{1}{1+x^2} = 1 - x^2 + x^4 - x^6 + \cdots + \left(-x^2\right)^n + \cdots$$
$$\frac{2x}{1+x^2} = 2x - 2x^3 + 2x^5 - 2x^7 + \cdots + (-1)^n 2x^{2n+1} + \cdots$$

$3 : \begin{cases} 1 : \text{two of the first four terms} \\ 1 : \text{remaining terms} \\ 1 : \text{general term} \end{cases}$

(b) No, the series does not converge when $x = 1$ because when $x = 1$, the terms of the series do not converge to 0.

1 : answer with reason

(c)
$$\ln\left(1+x^2\right) = \int_0^x \frac{2t}{1+t^2}\,dt$$
$$= \int_0^x \left(2t - 2t^3 + 2t^5 - 2t^7 + \cdots\right) dt$$
$$= x^2 - \frac{1}{2}x^4 + \frac{1}{3}x^6 - \frac{1}{4}x^8 + \cdots$$

$2 : \begin{cases} 1 : \text{two of the first four terms} \\ 1 : \text{remaining terms} \end{cases}$

(d)
$$\ln\left(\frac{5}{4}\right) = \ln\left(1 + \frac{1}{4}\right) = \left(\frac{1}{2}\right)^2 - \frac{1}{2}\left(\frac{1}{2}\right)^4 + \frac{1}{3}\left(\frac{1}{2}\right)^6 - \frac{1}{4}\left(\frac{1}{2}\right)^8 + \cdots$$

Let $A = \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^4 = \frac{7}{32}$.

Since the series is a converging alternating series and the absolute values of the individual terms decrease to 0,

$$\left|A - \ln\left(\frac{5}{4}\right)\right| < \left|\frac{1}{3}\left(\frac{1}{2}\right)^6\right| = \frac{1}{3} \cdot \frac{1}{64} < \frac{1}{100}.$$

$3 : \begin{cases} 1 : \text{uses } x = \frac{1}{2} \\ 1 : \text{value of } A \\ 1 : \text{justification} \end{cases}$

Form B
BC 6
6A$_1$

Work for problem 6(a)

$f(x) = 2x \cdot \frac{1}{1+x^2}$

$= 2x\,(1 - x^2 + x^4 - x^6 + \cdots)$ $\quad$ ~~$-1<x^2<1$~~

$= 2x - 2x^3 + 2x^5 - 2x^7 + \cdots$ ~~$+2x^{2n+1}$~~ $\quad$ ~~$-1<x<1$~~

$+ (-1)^n\, 2x^{2n+1}$

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Work for problem 6(b)

$f(x) = \sum_{n=0}^{\infty} (-1)^n\, 2x^{2n+1}$

$\rho = \lim_{n\to\infty} \left| \frac{2x^{2n+3}}{2x^{2n+1}} \right| = \lim_{n\to\infty} |x^2| < 1$ For the series to converge

$-1 < x < 1$

when $x=1$

series $\sum_{n=0}^{\infty} (-1)^n\, 2$ $\quad$ $\lim_{n\to\infty} |(-1)^n 2| = \lim_{n\to\infty} 2 \neq 0$

series diverges by divergent test

the series does not converge to $f(1)$ at $x=1$

Continue problem 6 on page 15.

NO CALCULATOR ALLOWED

Work for problem 6(c)

$f(x) = \frac{2x}{1+x^2} = 2x - 2x^3 + 2x^5 - 2x^7 + \cdots + (-1)^n 2x^{2n+1}$

$\ln(1+x^2) = \int_0^x f(t)\,dt$

$= x^2 - \frac{x^4}{2} + \frac{x^6}{3} - \frac{x^8}{4} + \cdots \quad -1 < x < 1$

Work for problem 6(d)

$\ln(\frac{5}{4}) = \ln(1+x^2) \quad x = \frac{1}{2}$

For $\ln(1+x^2) = x^2 - \frac{x^4}{2} + \frac{x^6}{3} - \frac{x^8}{4} + \cdots + \frac{(-1)^{n+1} x^{2n}}{n}$

It is a alternating series.

Also for $-1 < x < 1$

$|a_n| > |a_{n+1}|$

$\lim_{n \to \infty} |a_n| = 0 \quad$ for $-1 < x < 1$

$\therefore$ The alternating series converges by the alternating series test.

$|A - \ln(\frac{5}{4})| < \frac{1}{100}$ when $n = 2 \quad a_{n+1} = \frac{(\frac{1}{2})^6}{3} = \frac{1}{192} < \frac{1}{100}$

$\therefore A = (\frac{1}{2})^2 - \frac{(\frac{1}{2})^4}{2} = \frac{1}{4} - \frac{1}{32} = \frac{7}{32}$ //.

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Form B
BC 6
6B₁

NO CALCULATOR ALLOWED

Work for problem 6(a)

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \ldots + x^n$$

$$\frac{1}{1+x^2} = 1 - x^2 + x^4 - x^6 + \ldots + (-1)^n x^{2n}$$

$$\frac{2x}{1+x^2} = 2x - 2x^3 + 2x^5 - 2x^7 + 2x^9 + (-1)^n 2x(x^{2n}) + \ldots$$

Work for problem 6(b)

Integral test

Ratio Test

$$\rho = \frac{t_{n+1}}{t_n} = \frac{(-1)^{\cancel{n+1}} \cancel{2x}(x^{\cancel{2n+2}})}{\cancel{(-1)^n} \cancel{2x}(x^{\cancel{2n}})} = -1(x^2)$$

$$\lim_{n\to\infty}|\rho| = \lim_{n\to\infty} x^2 = x^2 \quad \text{when } x=1,\ x^2=1$$

yes because the function of $f(x)$ when integrated converges to $f(1)$ when $x=1$.

Continue problem 6 on page 15.

NO CALCULATOR ALLOWED

Work for problem 6(c)

$$\ln(1+x^2) = \int \frac{2x}{1+x^2}dx = \int 2x - \int 2x^3 + \int 2x^5 - \int 2x^7 + \ldots$$

$$\ln(1+x^2) = x^2 - \frac{2x^4}{4} + \frac{2x^6}{6} - \frac{2x^8}{8} + \frac{2x^{10}}{10} + \ldots$$

Work for problem 6(d)

$1+x^2 = \frac{5}{4}$ when $x = \frac{1}{2}$.

evaluate $\ln(1+x^2)$ when $x = \frac{1}{2}$

$= \frac{1}{4} - \frac{2}{4}\left(\frac{1}{2}\right)^4 + \frac{2}{6}\left(\frac{1}{2}\right)^6 - \frac{2}{8}\left(\frac{1}{2}\right)^8 + \ldots$

= value it approaches (lets call it F)

$\frac{1}{100} - F < A < \frac{1}{100} + F$

Because the value it approaches is the value of $\ln(5/4)$, then you can find A as an interval between $\frac{1}{100} - F < A < \frac{1}{100} + F$. Since $\frac{1}{100} - F$ & $\frac{1}{100} + F$ are decimals, the rational # A in that interval can then be determined.

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FORM B BC6 6C1

NO CALCULATOR ALLOWED

Work for problem 6(a)

$f(x) = \frac{1}{1+x}$ $f(0) = 1$ $1 - x + \frac{2x^2}{2!} - \frac{6x^3}{3!} \dots$

$f'(x) = -\frac{1}{(1+x)^2}$ $f'(0) = -1$ $1 - x + x^2 - x^3 \dots (-1)^n (x)^n$

$f''(x) = \frac{2}{(1+x)^3}$ $f''(0) = 2$

$f'''(x) = -\frac{6}{(1+x)^4}$ $f'''(0) = -6$

For $f(x) = \frac{2x}{1+x^2}$

$1 - x^2 + x^4 - x^6 \dots (-1)^n (x)^{2n+1}$

$2x - 2x^3 + 2x^5 - 2x^7 \dots (-1)^n (x)^{2n+1}(2)$

$\therefore \boxed{2x - 2x^3 + 2x^5 - 2x^7 \dots (-1)^n 2x^{2n+1}}$

Work for problem 6(b)

$\sum_{n=0}^{\infty} (-1)^n 2x^{2n+1}$

at $x = 1$

$\sum (-1)^n \cdot 2$

This series will not converge to $f(1)$. This is an alternating series. For it to converge, its limit as it approaches ∞ must be zero. This is not the case. The values of the sequence will oscillate between -2 and 2.

Continue problem 6 on page 15.

Work for problem 6(c)

$f'(x) = \frac{2x}{1+x^2}$ $\quad \int f'(x) = \ln(1+x^2)$

$$\int 2x - 2x^3 + 2x^5 - 2x^7 \ldots (-1)^n(2)(x^{2n+1})\,dx$$

$$= \boxed{x^2 - \frac{2x^4}{5} + \frac{2x^6}{7} - \frac{2x^8}{8} \ldots \frac{(-1)^n 2x^{2n+2}}{2n+2}}$$

Work for problem 6(d)

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Do not write beyond this border.

Question 6

Sample: 6A
Score: 9

The student earned all 9 points.

Sample: 6B
Score: 6

The student earned 6 points: 3 points in part (a), no points in part (b), 2 points in part (c), and 1 point in part (d). The student presents correct work in parts (a) and (c). In part (b) the student appeals to two different tests for convergence but makes an incorrect conclusion that the series "converges to $f(1)$." In part (d) the student correctly uses $x = \frac{1}{2}$ but does not find a rational number A, so the last 2 points were not earned.

Sample: 6C
Score: 4

The student earned 4 points: 3 points in part (a), 1 point in part (b), no points in part (c), and no points in part (d). The student presents correct work in parts (a) and (b). In part (c) the student antidifferentiates the first term from part (a) correctly but does not antidifferentiate the other terms correctly, so no points were earned.